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A General Equilibrium Approach to Carbon Permit Banking.

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A GENERAL EQUILIBRIUM APPROACH TO CARBON PERMIT BANKING

LOICK DUBOIS JEAN-GUILLAUME SAHUC GAUTHIER VERMANDEL

ABSTRACT. We study the general equilibrium effects of carbon permit banking during the transition to a climate-neutral economy by 2050. To this end, we develop and estimate an environmental real business cycle model for the European Union, in which the business sector is regulated by an emission trading system. Firms are allowed to transfer unused permits from one period to the next (banking), but the reverse direction (borrowing) is prohibited. Allowing for positive banking gives firms the opportunity to act as speculators and enables them to smooth their permit demand along the business cycle. Our projection exercises underscore the critical role of permit banking in shaping the policy outcomes. Specifically, the 2023 cap reform would lead to a strong increase in permit banking until 2035, a doubling of the carbon price, and an average GDP loss of approximately 5.3% or 6% (depending on whether we account for the market stability reserve) by 2060. Importantly, forgetting about permit banking when assessing cap policies would lead to both a significant underestimation of the total macroeconomic effects and an inaccurate representation of the carbon emission trajectory.

JEL: C32, E32, Q50, Q52, Q58.

Keywords: Emission trading systems, cap policies, carbon permit banking, environmental real business cycle model, occasionally-binding constraints, nonlinear estimation

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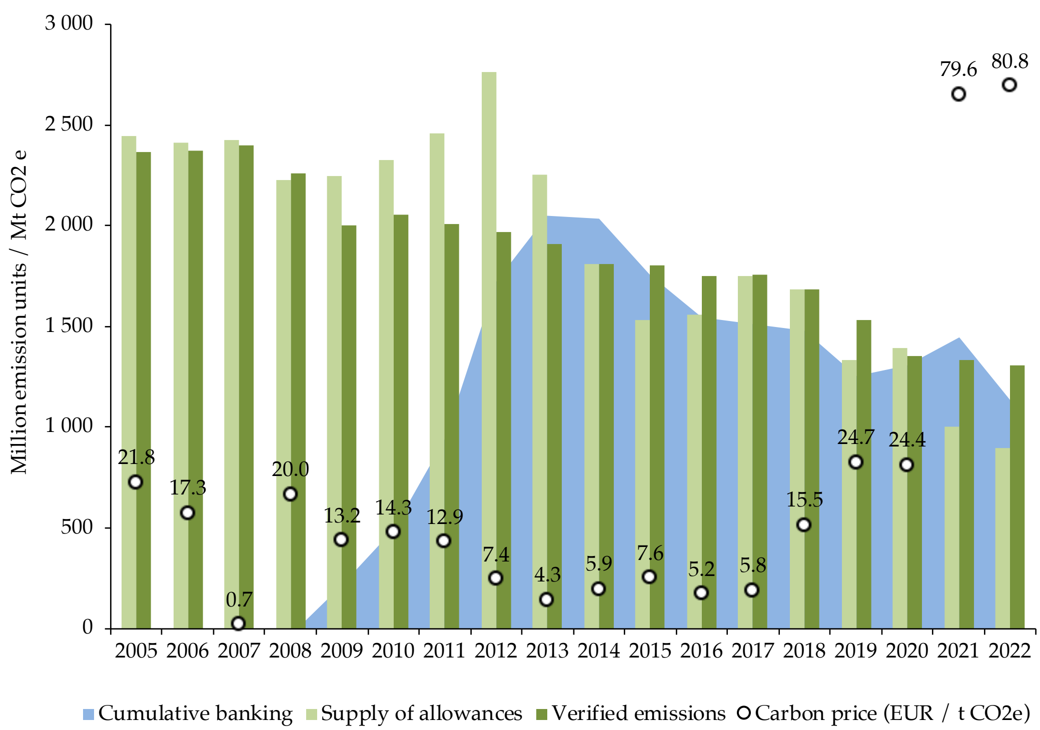
January 2024. L. Dubois: University Paris-Dauphine, LEDA UMR CNRS 8007, Place du Maréchal de Lattre de Tassigny, 75016 Paris, France (e-mail: loick.dubois@dauphine.fr). J.-G. Sahuc: Banque de France, 31 rue Croix des Petits Champs, 75049 Paris, France, and University Paris-Nanterre (e-mail: jean-guillaume.sahuc@banque-france.fr). G. Vermandel: Ecole Polytechnique, CMAP UMR CNRS 7641, Route de Saclay, Palaiseau and University Paris-Dauphine, LEDA UMR CNRS 8007, Place du Maréchal de Lattre de Tassigny, 75016 Paris, France (e-mail: gauthier@vermandel.fr). We thank Garth Heutel for valuable comments. The views expressed in this paper are those of the authors and do not necessarily reflect the views of the Banque de France or the Eurosystem. Declarations of interest: None.

1. INTRODUCTION

A carbon cap policy, also known as an emission trading system (ETS), is a market-based approach implemented by regulators to adjust and reduce carbon emissions. It was designed to address the negative externalities associated with greenhouse gas emissions, particularly carbon dioxide emissions, which contribute to climate change. Under a cap policy, the regulator sets a limit, or "cap", on the total amount of emissions allowed within a specific jurisdiction or industry. This emission cap is usually expressed in terms of a specific number of permits, with each permit representing the right to emit a certain amount of carbon dioxide or other greenhouse gases. In such a system, companies can strategically manage their emissions over time through *permit banking*. Firms that can reduce their emissions below the level implied by their allocated permits can bank surplus permits for future use or trade them to other entities. Figure 1 shows that cumulative banking (i.e., the total number of permits in circulation) in the European Union (EU)-ETS represents considerable amounts (blue area). It reached almost 2.1 billion at its peak in 2013 before decreasing to 1.1 billion in 2022, which is higher than the same year's worth of market supply.

FIGURE 1. Carbon emissions, allowances, banking and prices in the EU-ETS



Sources: European Environment Agency and European Commission. Cumulative banking is defined as the difference between the allowances allocated for free, auctioned or sold plus international credits surrendered or exchanged from 2008 to 2022 minus the cumulative emissions. MtCO2e: million tons of CO2 equivalent.

In the coming years, the EU-ETS is expected to play a critical role in the Union's efforts to reach the goals of the Paris Agreement. A significant portion of the reduction in carbon emissions to zero by 2050 must be driven by a decrease in permit supply. Nevertheless, considerable uncertainty remains concerning the future cap trajectories and the way companies adjust their permit banking strategies and production processes, ultimately affecting overall economic costs. Thus, a comprehensive economic assessment is crucial in designing and implementing effective environmental policies, as it enables policymakers to strike a balance between environmental targets and economic growth.

In this paper, we propose a general equilibrium approach to carbon permit banking and assess the macroeconomic effects of various cap policies within this context. Permit banking gives firms the opportunity to act as speculators and enables them to smooth their demand for permits along the business cycle. By spreading out emissions reduction and compliance costs, firms can minimize fluctuations in their marginal costs and maintain their production levels. Effective permit banking can thus contribute to market stability, reduce uncertainty and encourage consumer spending and business investments. Nevertheless, the general equilibrium effects of permit banking are highly dependent on both its aggregate level (high in the EU as Figure 1 shows) and the stringency of the underlying cap policies. This means that, to provide a practical quantification that is relevant to policymakers, the evaluation of these effects must be conducted under real-world conditions, in light of official regulations.

This study brings three main contributions. First, we develop an *environmental real business cycle* (E-RBC) model that embeds an emission trading system with permit banking. The model includes *(i)* households that maximize intertemporal utility by choosing consumption, hours worked and capital accumulation, and *(ii)* firms that produce a homogeneous final good, which could in turn be used for consumption and investment. As firms' activities generate CO_2 emissions, regulatory authorities implement an emission cap policy that grants them legal authorization to release a specified quantity. This allocated amount is contingent on the number of pollution permits issued by regulators. We allow firms that purchase carbon emission permits to either use them directly or bank them for later use, without the possibility of borrowing between allowance periods. This non-borrowing constraint captures the realistic dynamics of intertemporal permit trading and generates additional forward-looking dynamics that impact firm behavior. Finally, we assume that firms can reduce their carbon emissions by conducting costly abatement activities. The resulting model has appealing properties that make it amenable to the analysis of alternative economic policies as well as to empirical testing or validation. Specifically, it *(i)* formalizes the behavior of economic agents based on explicit microfoundations, *(ii)* manages all interactions between them within general equilibrium, *(iii)* emulates how forward-looking agents form expectations about a future characterized by stochastic events or outcomes, and *(iv)* incorporates uncertainty into agents' decision-making processes, as suggested by Pindyck (2013). Crucially, this framework effectively addresses the impacts of regulatory measures when firms rely on expectations to determine the optimal use of allowances in the future, akin to the principles derived from the Hotelling (1931)'s rule.

Our second contribution is the *estimation of this nonlinear model* by applying full-information methods to monthly EU data. The presence of an occasionally-binding constraint, arising from the non-negativity of permit banking, breaks the linear assumption commonly used in the literature on estimated structural models (e.g., Smets and Wouters, 2007). Our chosen piecewise-linear solution method handles the constraint as two different regimes in which it is either slack or binding (Guerrieri and Iacoviello, 2015; Aruoba et al., 2021). A recursive representation of the solution is obtained, conditional on how long the constraint is expected to bind in the future. Once a set of measurement equations is specified to link the state variables to the observables, an inversion filter can be used to compute the likelihood function analytically by inverting the observation equations to compute structural shocks (Guerrieri and Iacoviello, 2017; Cuba-Borda et al., 2019). Bringing the model to the data underscores the critical role of accounting for permit banking in understanding business cycle fluctuations. Notably, plugging the estimated shocks into an alternative version without permit banking results in excessive volatility for most variables. This occurs because, without the ability to store permits, firms struggle to insure themselves against future economic disruptions and to smoothly adapt their production processes. This empirical confrontation reveals that overlooking the intertemporal banking of permits would steer a policymaker toward an inappropriate representation of the economy.

Our third contribution is to propose several *projection exercises* based on recent decisions or regulations on the ETS, already implemented or announced by the European Parliament. These simulations assess how permit banking affects the macroeconomic outcomes of cap policies. Six major findings emerge from these exercises. First, applying the new linear reduction factor sequence to the cap for stationary installations (4.3% from 2024 to 2027 and 4.4% from 2028 onward) leads to a strong increase in permit banking until 2035 (before fading after that date), a doubling of the carbon price, and an average output loss of 6% by 2060. Second, policymakers can achieve the emission reduction path obtained with a cap policy through a tax policy by setting a path for the carbon price that accounts for firms' forward-looking behavior. This choice allows her to lower the output loss by 1.3% on average until 2060. Third, forgetting about permit banking leads to a significant underestimation of the macroeconomic effects of policy tightening, and an incorrect emission path. The latter misleadingly suggests that achieving net-zero emissions would occur by 2040. Fourth, announcing a policy in advance allows agents to modify their behavior accordingly, thus reducing emissions from the day of the announcement and not only by the time of implementation. Fifth, a frontloading of permits (as announced in February 2023) results in a net drop in emissions during the implementation period but after an increase in the stock of pollution in the atmosphere, a net negative effect on the carbon price and a reduction in GDP over both periods of frontloading and withdrawal. Finally, the market stability reserve, which triggers adjustments to the annual auction volumes if requirements based on the level of the aggregate bank of allowances are met, is a powerful tool that slows down firms' banking of permits and thus reduces emissions more quickly. By combining it with a higher linear reduction factor (e.g, 10%), the net zero objective would be achieved in 2050, with an average GDP cost of approximately 5.3%.

Our paper is related to the literature on permit banking, which generally recognizes its importance in achieving cost-effective emissions reductions and providing flexibility to regulated entities. In particular, Cronshaw and Kruse (1996), Rubin (1996), and Schennach (2000) show that the price of allowances should rise at the same rate as the real interest rate, which is consistent with Hotelling (1931). As the cost of emissions might increase over time owing to more stringent environmental regulations, firms can strategically use their banked permits to offset higher future costs. Recent contributions have extended these early studies in several directions, such as *(i)* the role of firms' market power (Liski and Montero, 2005), *(ii)* policy choices under uncertainty (Fell and Morgenstern, 2010; Fell et al., 2012), *(iii)* the role of delayed compliance (Holland and Moore, 2013), *(iv)* the interaction of the ETS with the electricity market (Pommeret and Schubert, 2018), *(v)* the proposal of various ETS stabilization mechanisms (Kollenberg and Taschini, 2016, 2019; Lintunen and Kuusela, 2018), *(vi)* putting into perspective the common features between pricing of allowances and financial claims (Hitzemann and Uhrig-Homburg, 2018; Jaccard et al., 2023), and *(vii)* the introduction of the market stability reserve (Perino and Willner, 2016; Quemin and Trotignon, 2021). However, these studies are interested in the role of banking in the functioning of the emissions market itself without paying significant attention to its broader effects on the economy. By contrast, we propose a general equilibrium approach to permit banking with a non-borrowing constraint that allows us to quantify the effects of cap policies on the real side of the economy. This nuanced view contributes to a deeper understanding of the interactions between permit banking and fundamental aspects of economic activity.

Our work also complements the burgeoning literature that focuses on climate issues using microfounded structural models. Fischer and Springborn (2011), Heutel (2012), and Angelopoulos et al. (2013) are among the first to introduce CO_2 emissions into real business cycle models. They assume that emissions stem from production and adversely impact utility, productivity, and output. Recent contributions have extended these models in several directions, including *(i)* multisector aspects (Golosov et al., 2014; Dissou and Karnizova, 2016), *(ii)* labor market frictions (Gibson and Heutel, 2020; Finkelstein Shapiro and Metcalf, 2023), *(iii)* distortionary fiscal policy (Barrage, 2020), *(iv)* endogenous entry (Annicchiarico et al., 2018; Finkelstein Shapiro and Metcalf, 2023), *(v)* public subsidies (Jondeau et al., 2023), and *(vi)* nominal rigidities and monetary policy (Annicchiarico and Di Dio, 2015; Annicchiarico and Di Dio, 2017; Carattini et al., 2021; Diluiso et al., 2021; Ferrari and Nispi Landi, 2023). These models have gained prominent attention in policy circles as they can be used to investigate the effects of environmental policies on aggregate variables in both short and medium terms. We contribute to this literature by offering a tractable framework that embeds an ETS with permit banking and estimate it using European data. By accounting for an additional regime in which firms are allowed to store permits, we generalize previous frameworks that represent a cap policy as a unique regime (e.g., Fischer and Springborn, 2011 and Heutel, 2012).

The remainder of this paper is organized as follows. Section 2 describes the E-RBC model used in this study. Section 3 reports on the data, estimation methodology, and parameter estimates. Section 4 discusses the dynamic properties of the proposed model. Section 5 proposes a series of policy exercises for quantifying the effects of environmental regulations on the European Union's economy in the presence of permit banking. Finally, Section 6 concludes the paper.

2. The Model

The economy is described by an environmental real business cycle model. There is a unit mass of atomistic, identical, and infinitely lived households that maximize intertemporal utility by choosing consumption, hours worked, and capital accumulation. On the production side, there is a unit mass of atomistic firms that hire labor services and physical capital to produce a homogeneous final good, which could in turn be used for consumption and investment. Firms' activities generate CO_2 emissions, and do not consider their effects on pollution and environmental damage. To force them to internalize this externality, a regulatory authority implements a cap policy. Specifically, this environmental policy gives firms the legal right to pollute a certain amount, which depends on the number of pollution permits issued by the regulator. These permits are bankable, that is, they can be stored for future use. Finally, we assume that firms can reduce their carbon emissions by conducting costly abatement activities.

2.1. **Household sector.** Each household indexed by $i \in [0,1]$ maximizes its sequence of present and future utility flows that depend positively on consumption $c_{i,t}$ and negatively on hours worked $n_{i,t}$:

$$\mathrm{E}_t \sum_{s=0}^{\infty} \beta^s \left\{ \frac{(c_{i,t+s} - \varphi c_{t+s-1})^{1-\sigma} - 1}{1-\sigma} - \chi \frac{n_{i,t+s}^{1+\nu}}{1+\nu} \right\}, \tag{1}$$

subject to the sequence of real budget constraints

$$c_{i,t} + x_{i,t} + \mathcal{A}_{i,t}^{x} \leq w_t n_{i,t} + d_{i,t} + r_{k,t} k_{i,t-1}, \tag{2}$$

where E_t denotes the mathematical expectation operator conditional on the information available at t, $\beta \in (0,1)$ is the subjective discount factor, φ captures external habit formation ("catching up with the Joneses"), σ is the inverse of the elasticity of substitution in consumption, $\nu > 0$ is the inverse of the Frisch labor supply elasticity, and χ is a scale parameter. Variable $x_{i,t}$ is investment, $d_{i,t}$ is the equity payout received from the ownership of firms, and w_t is the real wage. Physical capital $k_{i,t}$ is rented to the firm at the rental rate $r_{k,t}$. $\mathcal{A}_{i,t}^{x} = \frac{\psi}{2} \left(\frac{x_{i,t}}{x_{i,t-1}} - 1 \right)^2 x_{i,t-1}$ represents adjustment costs on investment, with $\psi > 0$. Physical capital accumulates according to

$$k_{i,t} = (1-\delta)\, k_{i,t-1} + x_{i,t}, \tag{3}$$

where $\delta \in [0,1]$ is the depreciation rate of capital.

The first-order conditions with respect to $c_{i,t}$, $n_{i,t}$, $x_{i,t}$, and $k_{i,t}$ are:

$$\lambda_{h,i,t} = (c_{i,t} - \varphi c_{t-1})^{-\sigma}, \tag{4}$$

$$w_t = \frac{\chi n_{i,t}^{\nu}}{\lambda_{h,i,t}}, \tag{5}$$

$$q_{i,t} = 1 + \psi \left(\frac{x_{i,t}}{x_{i,t-1}} - 1 \right) - \beta \mathrm{E}_t \left\{ \frac{\lambda_{h,i,t+1}}{\lambda_{h,i,t}} \frac{\psi}{2} \left(\left(\frac{x_{i,t+1}}{x_{i,t}} \right)^2 - 1 \right) \right\}, \tag{6}$$

$$q_{i,t} = \beta \mathrm{E}_t \left\{ \frac{\lambda_{h,i,t+1}}{\lambda_{h,i,t}} \left((1-\delta) q_{i,t+1} + r_{k,t+1} \right) \right\}. \tag{7}$$

where $\lambda_{h,i,t}$ is the Lagrangian multiplier associated with household i's budget constraint and $q_{i,t}$ is the relative price of capital $k_{i,t}$ (i.e., the marginal Tobin's Q).

2.2. **Business sector.**

2.2.1. *Technology.* Each firm indexed by $j \in [0,1]$ produces a homogenous good using the following production function:

$$y_{j,t} = \varepsilon_{a,t} k_{j,t-1}^{\alpha} n_{j,t}^{1-\alpha}, \tag{8}$$

where $\alpha \in (0,1)$ denotes the capital share, $k_{j,t}$ and $n_{j,t}$ denote the amounts of physical capital and labor services used by the firm respectively, and $\varepsilon_{a,t}$ is the total factor productivity shock common to all firms.

During its production process, a firm generates CO_2 emissions, denoted by $e_{j,t}$, which accumulate to increase the stock of pollutants in the air (Heutel, 2012):

$$e_{j,t} = \eta \left(1 - \mu_{j,t}\right) y_{j,t}^{1-\gamma}, \tag{9}$$

where $\mu_{j,t}$ represents the effort to abat emissions, $1-\gamma$ is the elasticity of emissions with respect to output, and η is a scale parameter.

However, firms do not consider the effects of their activities on pollution or environmental damage. As firms are atomistic, their marginal impact on total CO_2 emissions is zero. Therefore, individual emissions constitute a negative production externality, and the regulator forces firms to internalize them by implementing a cap policy.[1] Specifically, the regulator sets an emission cap and issues a quantity of emissions permits ϑ_t consistent with that cap. Firms must hold permits for every ton of CO_2 they emit. To this end, a firm may buy permits on a specific market, thus establishing the permit (or equivalently carbon) price $p_{e,t}$. Firms that can reduce their current emissions at a lower cost may bank any excess permits for latter use. Two important properties relative to the dynamics of permit banking are as follows:

[1]Our objectif is to focus on a cost-effectiveness analysis. Consequently, it is not necessary to explicitly represent the economic damage stemming from the accumulation of pollutants in the atmosphere. Introducing an endogenous determination of the cap would be feasible; this policy would be significantly different from the cap policy currently implemented in Europe. Because our approach is mainly positive, we leave normative evaluation for future research.

Assumption 1. *The law of motion of firm j's bank of permits $b_{j,t}$ is given by:*

$$b_{j,t} = b_{j,t-1} + \vartheta_{j,t} - e_{j,t}. \tag{10}$$

This equation states that the current stock of permits is the sum of the previous period bank $b_{j,t-1}$ and the newly bought permits $\vartheta_{j,t}$ minus the number of surrendered permits, measured in terms of emissions unit $e_{j,t}$.

Assumption 2. *Firms are not allowed to borrow permits from the future, such that:*

$$b_{j,t} \geq 0. \tag{11}$$

This non-borrowing constraint is a crucial feature of the model that differentiates it from other general equilibrium models that study emission cap policies. It generalizes the way firms are required to comply with environmental policy by introducing nonlinear effects into firms' profit optimization problems. When $b_{j,t} = 0$, the model is isomorphic to the standard linear versions used in the literature, e.g., Fischer and Springborn (2011) or Heutel (2012): each period, firms would buy $\vartheta_{j,t} = e_{j,t}$ permits to make up for their contemporaneous emissions. However, allowing for positive banking gives firms the opportunity to act as speculators and enables them to smooth their permit demand along the business cycle.

Finally, firms may substitute carbon-intensive technologies with low-carbon technologies, but this change in the existing lines of production is costly. We assume that the cost of abatement technology (in proportion to output) is given by:

$$\mathcal{A}^{\mu}_{j,t} = \varepsilon_{\mu,t}\theta_1 \left[\mu^{\theta_2}_{j,t} + \frac{\kappa}{2}\left(\frac{\mu_{j,t}}{\mu_{j,t-1}} - 1\right)^2 \mu_{j,t-1} \right] y_{j,t}. \tag{12}$$

This expression embeds two costs, one structural and the other cyclical, both expressed in percentage of output, with θ_1 as in Nordhaus (2014) and $\varepsilon_{\mu,t}$ an abatement shock. The first term μ^{θ}_2 captures the long-term cost of reducing carbon emissions, as in DICE, where θ_2 is the abatement cost function curvature. The second quadratic term is an adjustment cost on abatement growth, with κ as a weighting parameter, that captures the cyclical adjustments of carbon emissions in response to a change in carbon price.

2.2.2. *Profits maximization.* The objective of a firm is to maximize its intertemporal profit:

$$\mathrm{E}_t \sum_{t=s}^{\infty} \Omega_{t,t+s} \{ y_{j,t+s} - w_{t+s} n_{j,t+s} - r_{k,t+s} k_{j,t+s-1} - p_{e,t+s} \vartheta_{j,t+s} - \mathcal{A}^{\mu}_{j,t+s} \}, \tag{13}$$

subject to Constraints (8)–(12). In this expression, $\Omega_{t,t+s} = \beta^s \frac{\lambda_{h,t+s}}{\lambda_{h,t}}$ is the stochastic discount factor that converts future payoffs into current values, and $\lambda_{h,t}$ is the Lagrange multiplier associated with the budget constraint of the representative household.

This problem yields the following first-order conditions for an optimal solution:

$$w_t = (1-\alpha)\frac{y_{j,t}}{n_{j,t}} mc_{j,t} \tag{14}$$

$$\mathrm{E}_t\left\{r^k_{t+1}\right\} = \mathrm{E}_t\left\{\alpha \frac{y_{j,t+1}}{k_{j,t}} mc_{j,t+1}\right\} \tag{15}$$

$$mc_{j,t} = 1 - \varepsilon_{\mu,t}\theta_1\left(\mu_{j,t}^{\theta_2} + \frac{\kappa}{2}\left(\frac{\mu_{j,t}}{\mu_{j,t-1}} - 1\right)^2 \mu_{j,t-1}\right) - \lambda_{f_1,j,t}(1-\gamma)\frac{e_{j,t}}{y_{j,t}} \tag{16}$$

$$\varepsilon_{\mu,t}\theta_1\left(\theta_2\mu_{j,t}^{\theta_2-1} + \kappa\left(\frac{\mu_{j,t}}{\mu_{j,t-1}} - 1\right)\right) y_{j,t} + \mathrm{E}_t\left\{\Omega_{t,t+1}\varepsilon_{\mu,t+1}\theta_1\frac{\kappa}{2}\left(1 - \left(\frac{\mu_{j,t+1}}{\mu_{j,t}}\right)^2\right) y_{j,t+1}\right\}, = \lambda_{f_1,j,t}\eta y_{j,t}^{1-\gamma} \tag{17}$$

$$\lambda_{f_1,j,t} = p_{e,t}, \tag{18}$$

$$\lambda_{f_1,j,t} = \mathrm{E}_t\left\{\Omega_{t,t+1}\lambda_{f_1,j,t+1}\right\} + \lambda_{f_2,j,t} \tag{19}$$

$$(\lambda_{f_2,j,t} = 0 \quad \text{and} \quad b_t \geq 0) \quad \text{or} \quad (\lambda_{f_2,j,t} > 0 \quad \text{and} \quad b_t = 0), \tag{20}$$

where $\lambda_{f_1,j,t}$ is the Lagrange multiplier associated with constraints (9) and (10), which have been combined into one, and $\lambda_{f_2,j,t}$ is the Lagrange multiplier associated with non-borrowing constraint (11), which is the shadow value of carbon emission permits.

Equation (14) is the first-order condition with respect to labor. It states that the real wage is equal to the marginal product of labor net of the marginal resources that must be spent on abatement and pollution permits. Indeed, as emissions are a by-product of output, any additional output creates the need for extra abatement and permits to comply with the cap policy.[2] Equation (15) represents the first-order condition with respect to capital, which indicates that the rental rate of capital is equal to its net marginal productivity. Equation (16) is the first-order condition with respect to output, which defines the marginal cost. Equation (17) is the first-order condition with respect to abatement, which equalizes the marginal benefits and costs of an additional abatement unit. This indicates the amount of resources a firm no longer needs to spend on purchasing permits. Equation (18) is the first-order condition with respect to the demand for new permits, which simply states that the Lagrangian multiplier $\lambda_{f_1,j,t}$ is equal to the price of a bankable carbon emission. Equation (19) is the first-order condition with respect to the bank of permits. It is a forward-looking equation that relates the contemporaneous carbon price to the discounted carbon price expectation and Lagrangian multiplier $\lambda_{f_2,j,t}$. Finally, Equation (20) is the Kuhn-Tucker slackness condition associated with the non-borrowing constraint.

2.2.3. *Implications of the non-borrowing constraint on permits.* Let us first look at the case $\lambda_{f_2,j,t} > 0$ and $b_t = 0$. Equation (19) indicates that the current permit price is above expectations for the next period. Therefore, firms have no incentive to bank permits because they expect to obtain them later at a cheaper price. Hence, banking does not occur: $b_t = 0$. When $\lambda_{f_2,j,t}$ tends toward zero, the current and expected prices become closer. Once $\lambda_{f_2,j,t}$ reaches zero, firms are indifferent to (i) buying a permit today for later use and (ii) buying it later. Banking can occur and b_t is allowed to be positive. Note that, because $\lambda_{f_2,j,t}$ is not negative, the current price is never below the expected price. If this were the case, an arbitrage opportunity would lead firms to buy an infinite number of permits and bank them. Instead, the banking opportunity creates an additional demand for permits at time t and contributes to increasing the current price to at least the value of $\mathrm{E}_t\{\Omega_{t,t+1} p_{e,t+1}\}$, consistently with the Hotelling principle. Thus, the economy can be in one of the following two regimes: (i) a regime without banking where the current price is higher than the future price of a permit, or (ii) a regime with banking where the two prices are equal. This characteristic introduces nonlinearity, which translates to the occasionally-binding constraint $\lambda_{f_2,j,t} \geq 0$.

2.3. **Regulatory authority.** To incentivize firms to reduce their emissions, a regulatory authority sets a cap $\bar{\vartheta}$ on the maximum level of emissions and creates permits for each unit of emissions allowed under the cap:

$$\vartheta_t = \varepsilon_{\vartheta,t} \bar{\vartheta}, \tag{21}$$

[2] If pollution was an input in the production function, this marginal cost component would disappear and an extra first-order condition relative to the optimal use of that input would appear.

where $\varepsilon_{\vartheta,t}$ is a shock that makes the effective permit supply time-varying.

2.4. **Market clearing and equilibrium conditions.** The aggregate resource constraint of the economy is obtained by integrating across households and firms:

$$\int_{j=0}^{1} y_{j,t}\mathrm{d}j = \int_{i=0}^{1}\int_{j=0}^{1} \left(c_{i,t} + x_{i,t} + p_{e,t}\vartheta_{j,t} + \mathcal{A}^{x}_{i,t} + \mathcal{A}^{\mu}_{j,t}\right) \mathrm{d}i\mathrm{d}j \tag{22}$$

Regarding the properties of the stochastic variables, all shocks follow an AR(1) process $\varepsilon_{x,t} = 1 - \rho_x + \rho_x \varepsilon_{x,t-1} + \zeta_{x,t}$, with $x \in \{a, \mu, \vartheta\}$. In all cases, $\zeta_{x,t} \sim i.i.d.\mathcal{N}\left(0, \sigma^2_x\right)$.

3. Inference

In this section, we estimate the general equilibrium model with permit banking by using the maximum-likelihood methodology. First, we describe how the nonlinear model is solved. We then detail the selected data and comment on the structural parameter estimates.

3.1. **Solving the model with an occasionally binding constraint.** The non-negativity constraint on the bank of permits introduces nonlinearity and creates de facto two regimes (see Equation (20)). Consequently, conventional linear methods that provide only a local approximation, cannot be used to solve the model. Thus, we rely on the piecewise linear perturbation approach proposed by Guerrieri and Iacoviello (2015), which is a variant of the extended perfect-foresight path method proposed by Fair and Taylor (1983). In a nutshell, the occasionally binding constraint can be handled as different regimes of the same model: under one regime, the occasionally binding constraint is slack, and under the other regime, the same constraint is binding. The model is first linearized around the non-stochastic steady state of one of the two regimes, chosen to be the "reference regime" (see Appendix A for details). This allows us to obtain a linear approximation of the decision rule under this regime. In our context, the reference regime is that in which $\lambda_{f2,j,t} \geq 0$ in Equation (20). When the constraint is evaluated as binding, the model switches regime. A "guess and verify" method is then used to retrieve the decision rule and determine how long the constraint will bind. The starting guess of the expected durations is based on a linear solution that ignores the constraint. A Newton-like algorithm *(i)* iterates backward until convergence to the reference regime to form a decision and *(ii)* verifies that the resulting decision rule is consistent with the guess. If required, a new guess is formulated and the same procedure is applied.

If the left-hand side of Equation (20) is chosen as the reference regime, then Equation (19) leads to $p_e = 0$ in the steady state. To avoid this situation, there should be no permit banking in the equilibrium. In addition, in the absence of shocks, the solution algorithm requires the model to converge back to the reference regime in finite time. This means that permit banking is transitory, although it can last for multiple successive periods. It eventually fades if no more shocks occur.

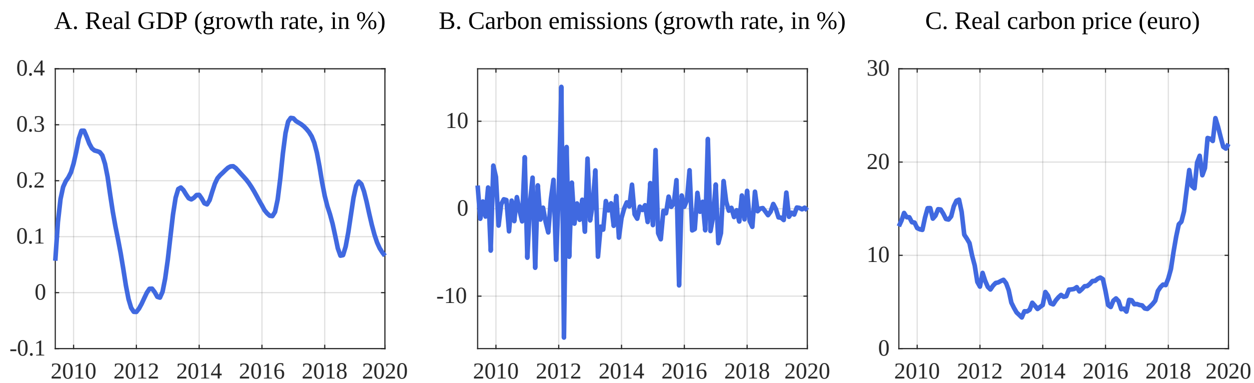
Importantly, this numerical approach generates a nonlinear state-space representation. Indeed, the dynamics in one of the two regimes may crucially depend on how long one expects to remain in that regime. The expected duration in this regime depends on the state vector. This interaction results in a high degree of nonlinearity.

3.2. **Data description.** The model is estimated using monthly data for the European Union from June 2009 to December 2019. Carbon emission data is taken from the Emissions Database for Global Atmospheric Research (Crippa et al., 2020), which provides estimates of global anthropogenic emissions and emissions trends, based on publicly available statistics (`https://data.jrc.ec.europa.eu/collection/edgar`). We build an aggregate time series of fossil CO2 emissions by summing the emissions of the 27 member countries of the European Union plus Iceland, Norway and the United Kingdom, the latter being part of the EU-ETS in our sample. The resulting series exhibits seasonal patterns. Thus, the data are seasonally adjusted using the X-13 ARIMA-SEATS filter from the Census Bureau (Lengwiller, 2022). The carbon price is obtained from the International Carbon Action Partnership (`https://icapcarbonaction.com/fr/node/839`), which offers a historical daily series, updated quarterly from the European Energy Exchange (the common auction platform of the EU-ETS designated by the European Commission). It is a spot price stemming from primary market auctions, i.e., the price at which permits are supplied directly from the government to firms to be either used directly, or banked for later use. The monthly time series is obtained by taking the average price for each month. Real GDP is taken from the OECD Main Economic Indicators, which offer a monthly proxy for OECD-Europe.[3] It is retrieved from the Federal Reserve of Saint Louis website (`https://fred.stlouisfed.org/series/OECDELORSGPORIXOBSAM`). Finally, we use the GDP deflator to construct a real carbon price, i.e., adjusted for the effects of price inflation. We extract the quarterly series of the GDP deflator for the European Union from the OECD Main Economic Indicators, retrieved from the Federal Reserve of Saint Louis website (`https://fred.stlouisfed.org/series/NAGIGP01EUQ661S`) and convert it into monthly data

[3] There is no monthly GDP series for the European Union. However, we found that the year-on-year GDP growth obtained from the monthly series (OECD-Europe) was very close to that of the official quarterly series for the European Union. Thus, they have the same business cycle characteristics.

using the Chow and Lin (1971) approach and the monthly consumer price index for OECD-Europe (https://fred.stlouisfed.org/series/OECDECPALTT01IXOBM). Figure 2 displays the retrieved variables used for the estimation.

FIGURE 2. Observable variables



Sources: Organization for Economic Cooperation and Development (GDP and deflator), Emissions Database for Global Atmospheric Research (carbon emissions), and International Carbon Action Partnership (carbon price).

The observable variable matrix is thus given by:

$$\begin{bmatrix} \text{Real GDP growth rate} \\ \text{Carbon emission growth rate} \\ \text{Real carbon price} \end{bmatrix} = 100 \times \begin{bmatrix} \Delta \log(y_t) \\ \Delta \log(e_t) \\ p_{e,t} \end{bmatrix}. \tag{23}$$

3.3. **Parameter values.** A first set of parameters is calibrated and is reported in Table 1. To be consistent with the monthly frequency, the discount rate β is set to 0.997, and the capital depreciation rate is set to 0.005 (i.e., an annual rate of 6%). The capital share in the production function is set to $1/3$ and the parameters $(\phi_1; \phi_2)$ associated with the abatement costs are $(0.1; 2.6)$, in line with Barrage and Nordhaus (2023).

TABLE 1. Calibrated parameters

	PARAMETER	VALUE
Discount factor	β	0.997
Capital depreciation rate	δ	0.005
Capital share of output	α	0.333
Abatement cost parameter (scale)	θ_1	0.100
Abatement cost parameter (elasticity)	θ_2	2.600

A second set of parameters is estimated using the full information maximum likelihood methodology. Specifically, we use an inversion filter to recursively extract shock innovations by inverting the observation equations conditional on an initial state. This approach allows for the easy computation of the likelihood function in the context of a model with an occasionally binding constraint (Guerrieri and Iacoviello, 2017; Kollmann, 2017). The last column of Table 2 reports the parameter estimates and their associated P-values.

TABLE 2. Estimated parameters

	PARAMETER	ESTIMATES
Panel A: Structural parameters		
Inv. of elasticity of substitution in consumption	σ	2.744 *[0.00]*
Inv. of Frisch labor supply elasticity	ν	1.927 *[0.00]*
Habit formation	φ	0.728 *[0.00]*
Elasticity of emissions with respect to output	$1-\gamma$	0.821 *[0.00]*
Abatement effort	μ	0.202 *[0.00]*
Adjustement cost on investment	ψ	5.926 *[0.00]*
Adjustement cost on abatement	κ	0.027 *[0.00]*
Panel B: Shock processes		
AR(1) productivity	ρ_a	0.949 *[0.00]*
AR(1) abatement cost	ρ_μ	0.908 *[0.00]*
AR(1) permit supply	ρ_ϑ	0.941 *[0.00]*
Std dev. productivity	σ_a	0.001 *[0.00]*
Std dev. abatement	σ_μ	0.221 *[0.00]*
Std dev. permit supply	σ_ϑ	0.031 *[0.00]*
Log likelihood		663.050

Note: P-values are in brackets (null hypothesis of being equal to zero).

All values are significant and consistent with those reported in the existing literature. In particular, usual parameters such as habit formation, elasticities in the utility function and adjustment costs on investment are close to those found in Smets and Wouters (2007). In addition, the elasticity of emissions with respect to output is estimated to be 0.82. This value lies in the interval (0.69–0.86) obtained by Heutel (2012) from regressions of the log of emissions on the log of GDP with three different data treatments (ARIMA, seasonally adjusted, and HP filters). The abatement effort is estimated to be 0.20. Combined with the values of θ_1 and θ_2, this estimates leads to steady-state abatement costs that amount to 0.15% of GDP, in line with Annicchiarico and Di Dio (2015). Furthermore, the value associated with abatement adjustment costs is relatively low (0.03), indicating that the model does not require additional smoothing for abatement. Finally, we estimate the parameters pertaining to the dynamics of the three shocks introduced in the model (ϵ_a, ϵ_μ, ϵ_ϑ). As is usually found in estimated dynamic stochastic general equilibrium models, shocks are highly autocorrelated (close to 0.9).

4. Dynamic properties of the model

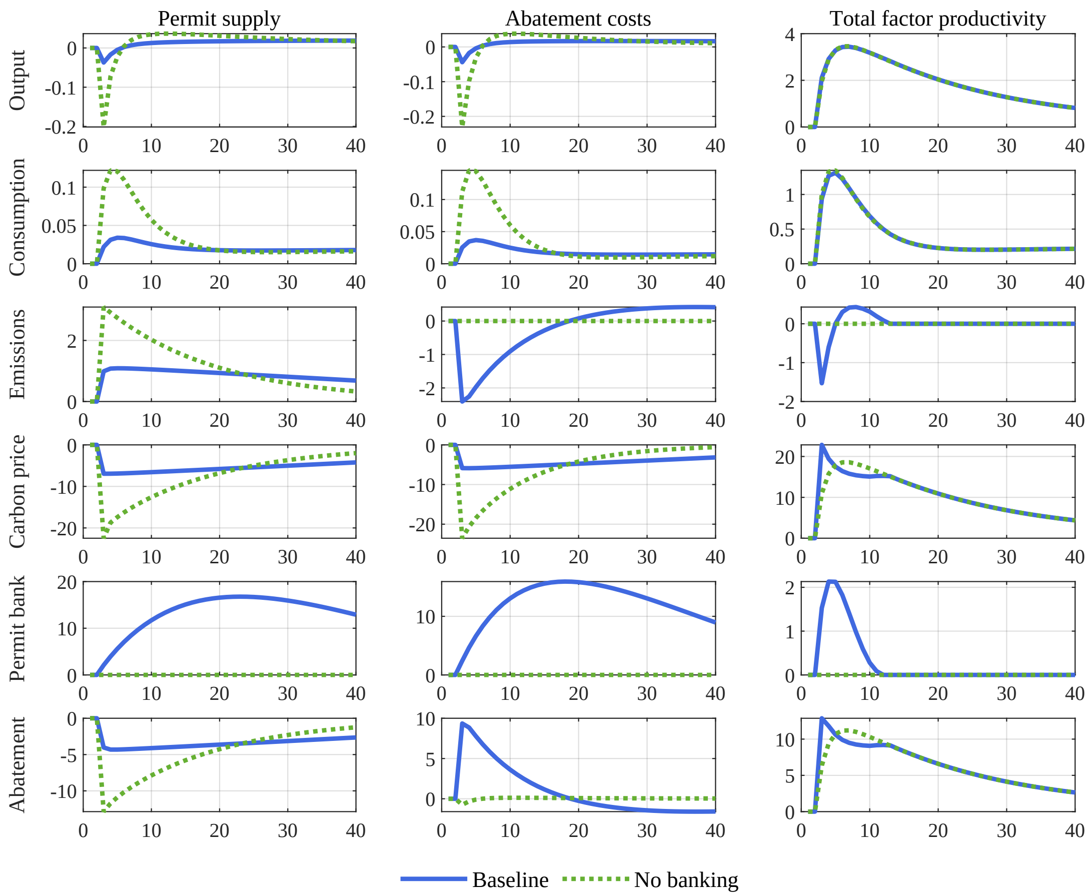
This section discusses the dynamic properties of the model through *(i)* the impulse response functions of a number of key variables to the three underlying shocks (total factor productivity, abatement costs, and permit supply) and *(ii)* a counterfactual exercise that provides insights into the effects of not taking into account permit banking.

4.1. **Impulse response functions.** Figure 3 displays the responses of the main macroeconomic and environmental variables to the three shocks embedded in the model for *(i)* the baseline model with permit banking (plain blue line) and *(ii)* an alternative version without banking (dotted green line). The latter is a linear version of the baseline model without permit banking. It is obtained by eliminating the lower bound on $\lambda_{f_2,t}$ and setting $b_t = 0$, which leads to $e_t = \vartheta_t$ at all times (cf. Equation (10)). Hence, impulse responses are expected to differ when the baseline model enters its second regime in which permit banking arises.

The first shock is a positive permit supply shock (first column). In both models, the shock increases carbon emissions and reduces carbon (or equivalently permit) price and abatement. However, in the baseline model, the temporarily reduced carbon price creates an incentive for firms to store permits. For a few periods after the impact, the reduction in the permit price in the baseline model is less pronounced than that in the model without banking. Banking opportunities create additional demand for permits in the short run. Abatement is also reduced less because of both the lower drop in the carbon price and the incentive for firms to fill the bank. In the medium run, when firms start to use banked permits, the demand for newly issued permits declines and the price stays below the path obtained in the model without banking. After firms have finished filling their reserves and start depleting them, carbon emissions remain at a higher level than in the no banking case for many periods. At this point, less abatement is needed for firms to comply with the policy.

The second shock is a negative shock to the abatement costs (second column). The reduction in abatement costs implies that fewer resources must be devoted to abatement goods in the economy, leading to a decrease in output (cf. the equilibrium resource constraint given by Equation (22)). Indeed, at general equilibrium, the resources used to abat emissions are

FIGURE 3. Impulse response functions



Note: The figure displays the impulse response functions (IRFs) of several variables to three shocks: permit supply (Column 1), abatement costs (Column 2), and total factor productivity (Column 3). Each IRF is expressed in percentage deviations from the steady-state, except for the bank of permits and carbon price.

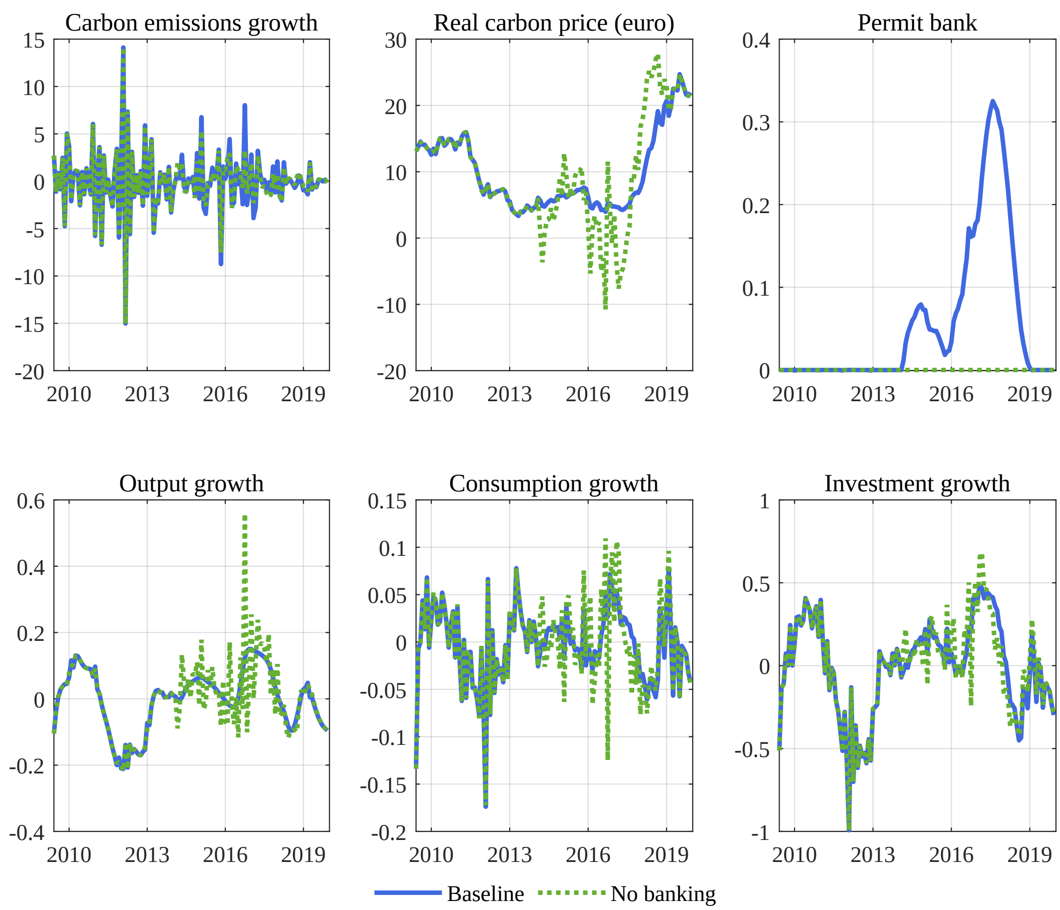
accounted for in production. Without banking, lower output for the same level of emissions implies less abatement. Hence, instead of allowing firms to produce more while conducting more abatement, reduced abatement costs allow them to produce less but more efficiently. This means that while total production decreases, the production net of abatement costs increases, as does consumption. Following this shock, the carbon price decreases, driven by *(i)* the reduced cost of the substitute of permits for compliance with the policy, and *(ii)* the lower demand for permits due to reduced production.

This shock is interesting because it puts forward a puzzle (we refer to it as the *abatement puzzle*) usually found in the literature, which can be solved by introducing a permit banking system. The standard environmental general equilibrium model unexpectedly predicts a reduction in abatement costs, yielding a reduction (of low amplitude in our example) in abatement efforts. This counterintuitive outcome originates from the restriction that emissions are always equal to the contemporaneous permit supply. Interestingly, the *abatement puzzle* is solved under permit banking because our model offers more intuitive abatement dynamics. While the general equilibrium effect described above is still at play, the banking channel modifies the behavior of firms. Lower abatement costs incentivize companies to immediately increase their abatement efforts, leaving them with the option of doing less later when costs rise. The carbon price is driven down but to a lesser extent than in the no-banking case, due to the additional demand for banking. To make the best use of the shock, firms increase abatement and fill their banks with the saved permits. Later, when firms use banked permits, abatement decreases more than it does in the no-banking model. The path of output is, therefore, modified with a lower loss at the beginning and then higher afterwards, when fewer resources are needed to conduct abatement and buy permits. The emission dynamics is no longer constrained to be the same as permit supply dynamics. Following the shock, increased abatement leads to lower emissions. Later, both the decrease in abatement and use of stored permits induce higher emissions. Thus, our model can reproduce the capacity of firms to smooth emissions along the business cycle.

The last shock is a positive disturbance to total factor productivity (third column). Unlike the two previous stochastic innovations, this shock is explicitly calibrated to have the constraint binding, to differentiate the dynamics of the two models. As emissions are a by-product of output, the shock automatically increases firms' pre-abatement pollution. This translates into increased demand for permits, higher carbon price, and higher required abatement effort. Banking opportunities create an additional dynamics in the baseline model. The realization of the shock and gradual capital accumulation cause peak productivity to materialize only after a few periods. Meanwhile, forward-looking firms begin building a bank of permits to be used during the most profitable times. Again, this additional demand for permits in the short run raises the carbon price relatively more than in the model without banking. Abatement also increases relatively more during the early stages. Once peak productivity is reached, firms start depleting the bank, and both the carbon price and abatement effort are lower than in the no-banking case until full depletion. Recall that emissions are allowed to differ from constant permit supply in our framework. They decrease early during the build-up of the bank and increase when the peak productivity is reached.

Overall, these impulse response functions illustrate the ability of the banking model to properly replicate the idea of Cronshaw and Kruse (1996): firms are willing to bank carbon permits when they expect that either the price will be higher later or that abatement will be costlier later compared to the current situation. The nonlinearity embedded in the model eliminates the restrictive assumption that emissions are equal to the contemporaneous permit supply at all times and adds more realism to the behavior of firms and the whole economy. Finally, the banking model can solve the *abatement puzzle* that typically appears in standard environmental models.

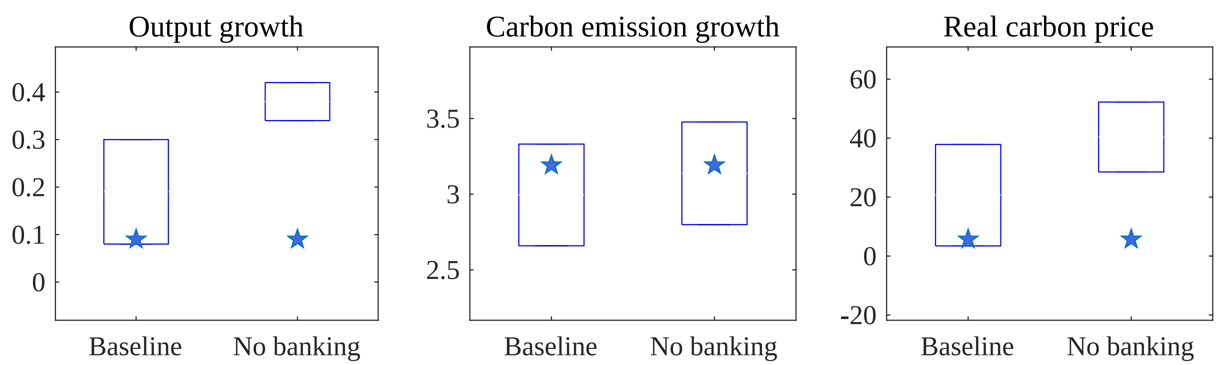
FIGURE 4. The counterfactual exercise



Note: The figure presents the trajectories of the main environmental and macroeconomic variables conditional on the estimated shocks, with and without (counterfactual) permit banking.

4.2. **The pitfall of assuming no permit banking.** In this section, we perform a counterfactual exercise to understand the importance of the nonlinearities generated by intertemporal banking of permits. It consists of plugging the smoothed shocks obtained from the estimated baseline model into an alternative version without permit banking. Figure 4 displays the results of this exercise (conditional on the estimated sequence of shocks from the baseline model), whereas Figure 5 reports the simulated second-order moments for each model version. First, a model without banking leads to higher volatility for most variables. This results in standard deviations of observable variables that are far beyond those of their empirical counterparts (except for emission growth), as shown in Figure 5. Firms that are not allowed to store permits are unable to insure themselves against fluctuations in permit supply, abatement costs and productivity. Emissions are always equal to the contemporaneous cap level, and the price of carbon is determined solely by the interaction between current permit supply and demand. Thus, any change in the cap level has an immediate and strong effect on the carbon price. This effect lasts only as long as the change in level does.

FIGURE 5. Empirical and model-implied standard errors



Note: The two models were simulated 300 times for 127 periods (same size as the data sample). The stars represent the values obtained from the data. The rectangles represent the range of values simulated from the baseline model.

Similarly, any change in permit demand (e.g., through shocks to productivity or abatement costs) directly affects the carbon price. In a version of the model without banking, these effects cannot be mitigated neither by additional demand for permits that could be used in the future nor by the firm already having a reserve of paid-for permits. This leads to more volatile carbon-price dynamics. When confronted with smoothed shocks, the alternative model predicts that this price would be negative at several points. This would mean that the ETS subsidizes the pollution. Indeed, in these instances the demand for permits is so much lower than the supply that the regulator pays for firms to maintain emissions at the cap level. In our baseline model, firms can reduce their emissions below this level and bank a surplus of permits for later use. This generates additional demand and maintains a positive carbon price. When shocks increase the carbon price, the latter also reaches greater heights if firms were not allowed to build a bank of permits. Increased carbon price volatility translates to increased macroeconomic volatility. Firms with no forward-looking abilities are highly dependent on the price they must pay to maintain their emissions at the level implied by the stringency of the current policy. Consequently, the volatility of output growth is predicted to be approximately four times higher than that found in the data (Figure 5). Likewise, consumption and investment are more volatile when permit banking is not accounted for.

This counterfactual exercise highlights that accounting for permit banking is crucial for correctly studying the interaction between a cap policy and the economy at the business cycle frequency.

5. Policy implications of permit banking

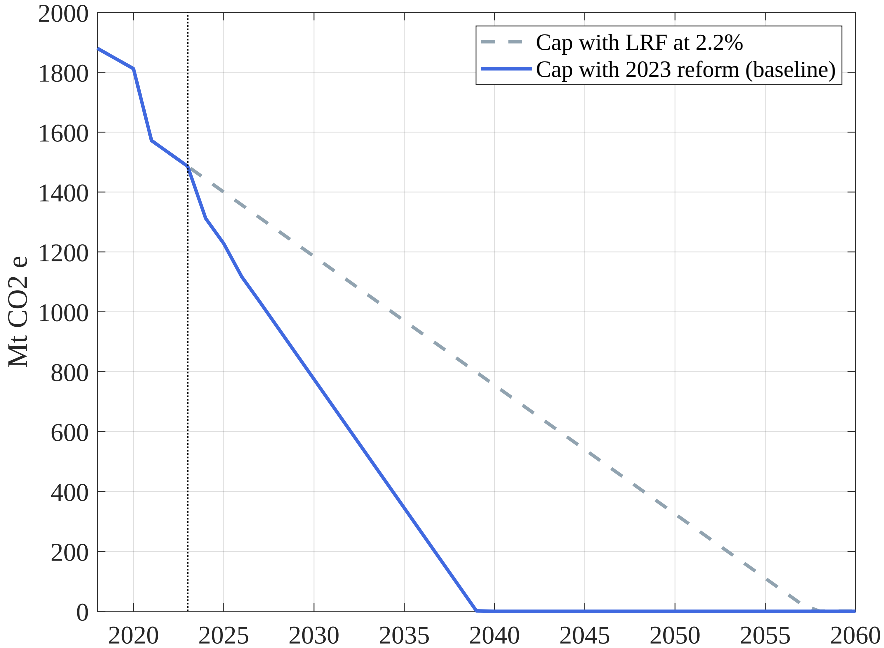
This section proposes a set of policy exercises for quantifying the effects of environmental regulations on the European Union's economy in the presence of permit banking. Specifically, we simulate the recent resolutions of the European Parliament associated with the emission trading system, which aim to change the overall cap on emissions and to propose new rules for auctioning and distributing emission allowances.[4]

5.1. **Baseline scenario.** An emission trading system requires that the cap it sets on carbon emissions would diminish over time to respect pre-determined climate goals. After setting a linear reduction factor (LRF) on the cap for stationary installations at 1.74% during ETS Phase 3 (2013-2020), the European Parliament announced in 2018 that from Phase 4 (2021-2030) onwards, the LRF would increase to 2.2% (Directive EU 2018/410). This path is represented by the dashed gray line in Figure 6. It was then realized that this LRF would not make it possible to reduce emissions by 55% by 2030 from 1990 levels (62% from 2005 levels), and thus would not be in line with the European Green Deal's emissions reduction targets. Consequently, it decided in May 2023, as part of the 'Fit for 55' package, to increase the LRF to 4.3% from 2024 to 2027 and to 4.4% from 2028 on (Directive (EU) 2023/959). It was further decided to apply two one-off cap reductions of 90 and 27 millions tons of CO2 equivalent (MtCO2e) in 2024 and 2026, respectively. These successive changes (present and future) constitute our *baseline scenario* for the 2023-2060 period (blue plain line in Figure 6).

To analyze the general equilibrium effects of such a decrease, we use our estimated model and run perfect foresight simulations, starting from the union-wide cap for stationary installations fixed at 1,529 million allowances in 2022. This type of simulations captures the fact

[4]The European Parliament, along with the Council of the European Union, shares the responsibility for adopting EU legislation, including policies related to climate change mitigation. It reviews, amends, and votes on proposals put forth by the European Commission, which form the basis of EU climate policy.

FIGURE 6. European-Union emission trading system cap

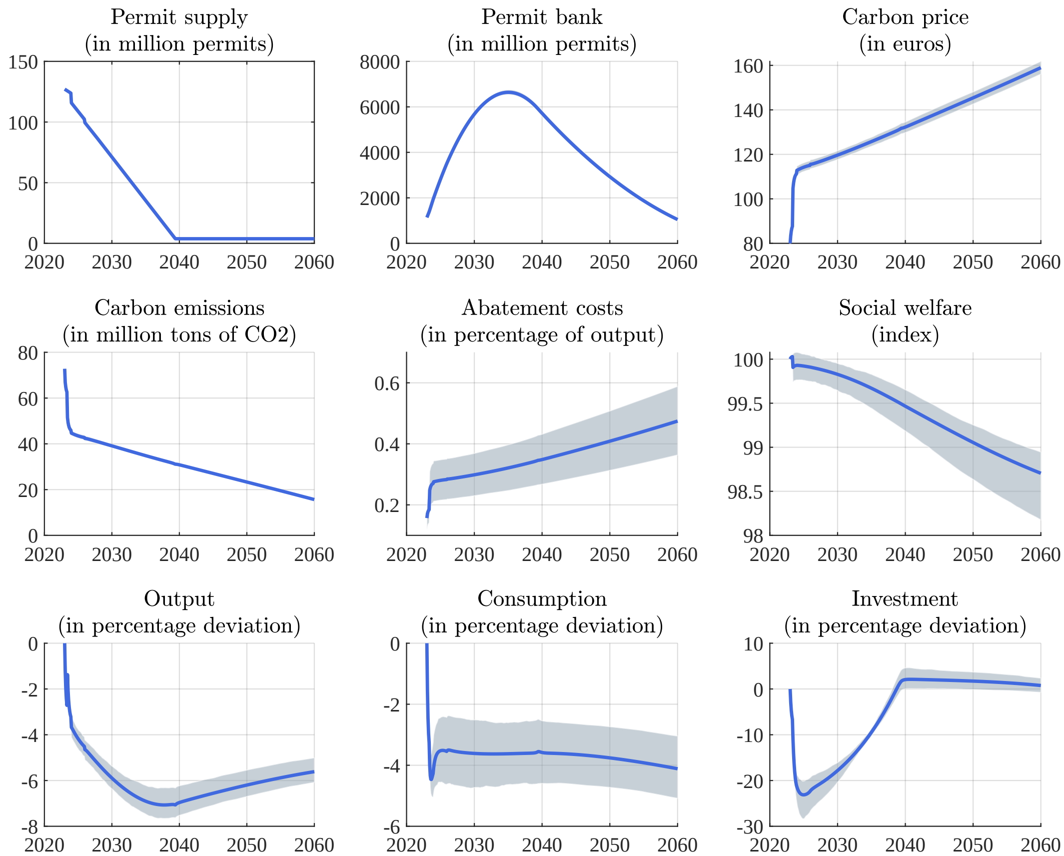


Note: The figure starts in 2018 where the cap was at 1,892 millions tonnes of CO2 equivalent (MtCO2e). A linear reduction factor of 1.74%, which translates into a year-on-year reduction of the cap by some 38 million allowances, is applied from 2018 to 2021. The cap is adjusted in 2021 to reflect the exit of the UK from the EU ETS. For the baseline scenario, we annually deduct 43 MtCO2e between 2021 and 2023 (LRF of 2.2%) and adjust this value to an LRF of 4.3% for 2024-2027 and of 4.4% from 2028 onward. In addition, a one-off reduction of 90 MtCO2e is applied in 2024, followed by a one-off reduction of 27 MtCO2e in 2026.

that regulatory changes are typically announced years in advance, leaving time for firms to adjust their expectations and behavior.

The results are shown in Figure 7. Following the announcement of a tightening of emission targets, the carbon price quickly raises from 80 euros to 115 euros to finally reach 160 euros in 2060. In anticipation of this gradual rise, firms bank a portion of the permits they purchase until they are obliged to use them to maintain their level of production, that is around 2035. Therefore, the dynamics of the bank of permits has a bell shape. The new targets force firms to abat more, implying a sharp increase in the total abatement costs. This phenomenon is reinforced early by firms' incentives to store permits. There is an abatement cost differential of 0.1% of output following the announcement, and almost 0.3% in 2060. This translates into an immediate 23% drop in detrended investment. It takes about fifteen years for this effect to dissipate. Detrended consumption also falls immediately and then remains persistently at a level lower by 3.5%. There is a 4% consumption gap by 2060. As a result, detrended total output (i.e., the sum of the demand component and all costs) suffers from an average

FIGURE 7. Baseline scenario under the 2023 EU-ETS cap reform



Note: This figure displays the trajectories of the main environmental and macroeconomic variables under the baseline scenario, which corresponds to the 2023 EU-ETS cap reform. Variables are expressed in monthly terms. Macroeconomic variables are detrended. Permit supply is assumed to be equal to the cap policy. The gray area represents the 68% confidence interval, based on 300 draws in a Normal distribution of the parameter estimates (computation time: 150 hours with a 3 GHz Intel Xeon W (10ï¿½cores) CPU processor).

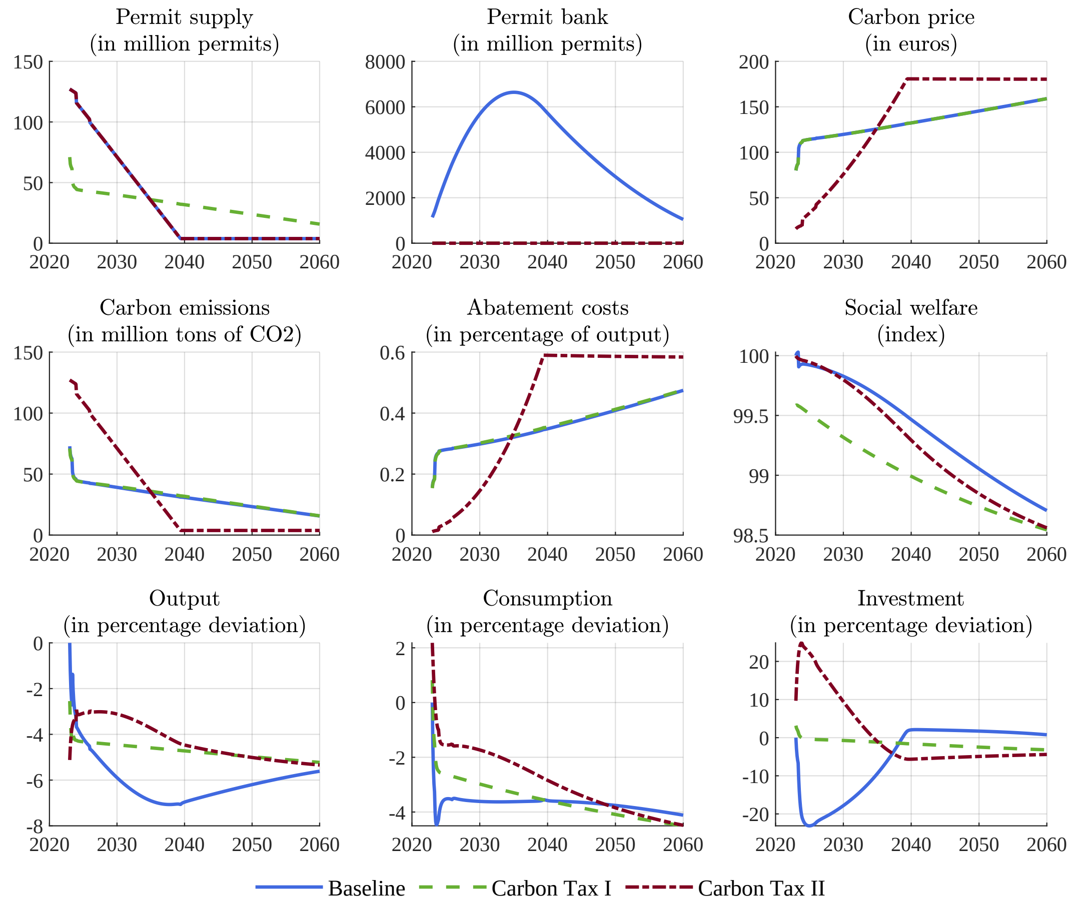
loss of approximately 6% by 2060. Furthermore, we observe that this scenario allows the European Union to achieve in 2030 the desired reduction by more than 62% of the emissions from their 2005 level (2,369 MtCO2e), but does not make it possible to reach net zero by 2050. From a normative perpective, social welfare (the discounted sum of utility over time) reduces under the cap reform as a result of the drop in production and the increased spending in abatement that diverts resources away from consumption. Because the model does not include the environmental gains from the policy, welfare reduces in response to it.

Result 1. *The baseline cap scenario leads to (i) a strong increase in permit banking until 2035 (before fading after that date), (ii) a doubling of the carbon price, and (iii) an average output loss of approximately 6% by 2060.*

5.2. **Cap policy versus carbon tax.** Before analyzing other decisions of the European Parliament, it is interesting to focus on the comparison between our cap policy with banking and a pure carbon tax policy. Indeed, comparisons of carbon regulation instruments in general equilibrium have thus far been limited to taxes versus intensity targets versus cap policies without banking (e.g., Fischer and Springborn, 2011, Annicchiarico and Di Dio, 2015, Annicchiarico and Diluiso, 2019). Furthermore, impact assessments of relevant policy actions typically focus on taxes (e.g., Shapiro and Metcalf, 2023). This section emphasizes the importance of using an appropriate framework for implementing simulation exercises. This implies accounting for the permit banking dimension when the current policy allows it. To do so, we compare our baseline cap decrease (i.e., with permit banking) developed in Section 5.1 to two different yet comparable tax scenarios. In a scenario called "Carbon Tax I", we set a tax that replicates the carbon price obtained in our baseline cap scenario. In a scenario called "Carbon Tax II", we set a tax that results in firms choosing to emit as much carbon as they would receive permits in the baseline scenario. In a business cycle setup where a tax sets a fixed carbon price and a cap (without banking) sets a fixed level of emissions, the two have different properties (Fischer and Springborn, 2011). However, it is important to note that, under certainty, for each increasing carbon tax, there is an equivalent decreasing cap (without banking), which goes back to the result of Weitzman (1974). Carbon tax II can thus be thought of as setting the increasing tax described earlier or as setting a cap at the same level and decreasing at the same rate as in the baseline scenario, but without allowing for permit banking. The results are shown in Figure 8.

The Carbon Tax I scenario yields results close to the baseline for the climate-related variables. In the latter, the opportunity to store permits increases the demand for new permits early and lowers it after 2035, when firms use their stored permits. This extra demand in turns relatively increases the carbon price early, before decreasing it. This information is stored in the Carbon Tax I's price. Now that firms only pay for the carbon that they emit immediately, they will first buy fewer permits than in the baseline case (and then more). In this way, they match the level of carbon emissions of firms subjected to the baseline cap at all times. The same carbon price implies the same abatement dynamics. As a result of firms buying fewer permits at the same price, Carbon Tax I leaves more room for early consumption and investment at the cost of a stronger future consumption loss. The absence of permit banking does not offer temporal flexibility for the private sector to adjust gradually to changes in the carbon price. Consequently, we observe a downward shift in the welfare curve due to consumption

FIGURE 8. Cap policy versus carbon tax



Note: This figure displays the trajectories of the main environmental and macroeconomic variables under alternative cap/carbon tax scenarios (in Carbon Tax I, a tax is set to replicate the carbon price obtained in the baseline cap scenario; in Carbon Tax II, a tax is set such that firms choose to emit as much carbon as they would receive permits in the baseline scenario). Variables are expressed in monthly terms. Macroeconomic variables are detrended. Permit supply is assumed to be equal to the cap policy.

(resp. hours worked) that is expected to be lower (resp. higher) than that of the baseline by 2060. The relative welfare gain of a cap policy over a carbon tax policy is consistent with Fischer and Springborn (2011), Annicchiarico and Di Dio (2015), and Annicchiarico et al. (2023), who show that an emission cap policy is likely to dampen macroeconomic fluctuations.

In contrast, Carbon Tax II's scenario implies significant deviations from the baseline for all the variables. Information from firms' forward-looking behavior disappears. Firms face a low carbon price early and emit a large amount of carbon immediately. The carbon price gradually increases so that, at all times, firms emit as much as they would be offered permits under the baseline scenario (equivalently, the cap decreases at the same rate as in the baseline scenario but does not allow for permit banking). This results in a significant frontloading of emissions. Firms' abatement follows the carbon price signal. A lower carbon price, lower abatement and higher emissions early entail a significant relative gain in consumption and investment until 2035. This gain is more important and lasts longer than that under Carbon Tax I, particularly because of the investment amplification mechanism. Consequently, social welfare is relatively higher than that of Carbon Tax I before gradually converging to the same level.

There are two important results that emerge from this exercise:

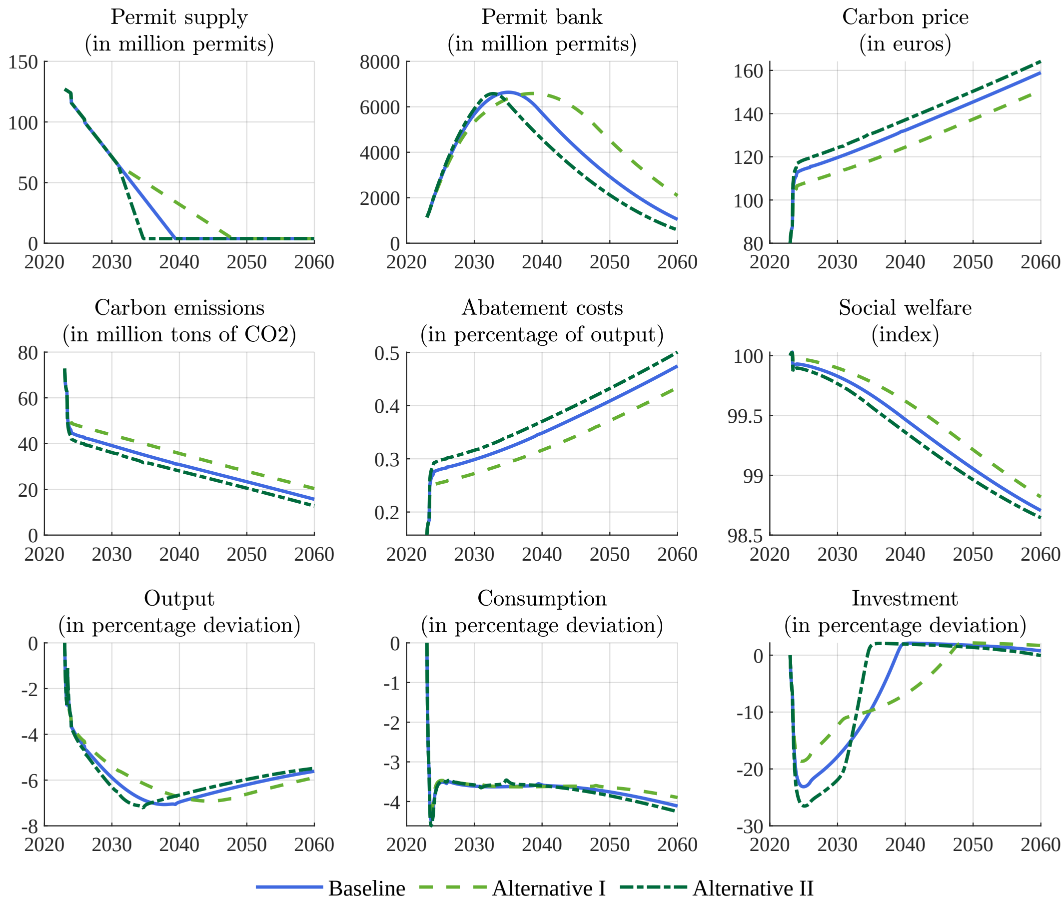
Result 2. *A policymaker can achieve the same emission reduction path as under cap policy by setting a carbon price that accounts for firms' forward-looking behavior implied by the ETS. This choice allows her to save 1.3% of GDP on average until 2060, at the cost of deteriorating social welfare.*

Result 3. *Forgetting permit banking leads to (i) a significant underestimation of the macroeconomic effects of policy tightening and (ii) an incorrect carbon emission path. The latter misleadingly suggests that achieving net-zero emissions would occur by 2040.*

5.3. **What about after 2030?** As indicated above, the European Parliament has announced trajectories for what is called Phase 4 of the EU-ETS (2021-2030). However, at this stage, there is no indication of the characteristics of Phase 5 which will begin in 2031. Therefore, there is much uncertainty regarding the future trajectories of permit supply. With respect to the baseline characterized by a 4.4% LRF after 2031, we propose two credible alternative scenarios that differ after this date. In a first scenario called "Alternative I", we decrease the LRF to 2.2% to return to a situation similar to that which prevailed in 2022. In the second scenario called "Alternative II", we increase the LRF to 10% to reach a cap of virtually 0 in 2035, to evaluate an early phase-out from carbon emissions.

Figure 9 shows the results for the three credible scenarios. As expected, decreasing the LRF is less restrictive for firms that reduce their emissions less and store their permits longer (green dashed line). This results in a smaller drop in GDP, at the cost of greater accumulation of CO_2 in the atmosphere. Conversely, increasing the LRF from 4.4% to 10% from 2031 allows the world to suffer fewer emissions, although not by a large amount compared to the baseline scenario, at the cost of a slightly greater GDP loss in the short term. Indeed, both consumption and investment would reach lower levels than those in the baseline scenario. In this context, firms can reduce their bank of permits more quickly and the carbon price is above that of the baseline case. Banking opportunities allow firms to spread the greater (smaller) cost of the transition implied by the increased (decreased) LRF over time and not suffer (benefit) from it sharply only from 2031 onward. Note also that, due to the presence of the permit bank, it is

FIGURE 9. Alternative cap scenarios from 2031

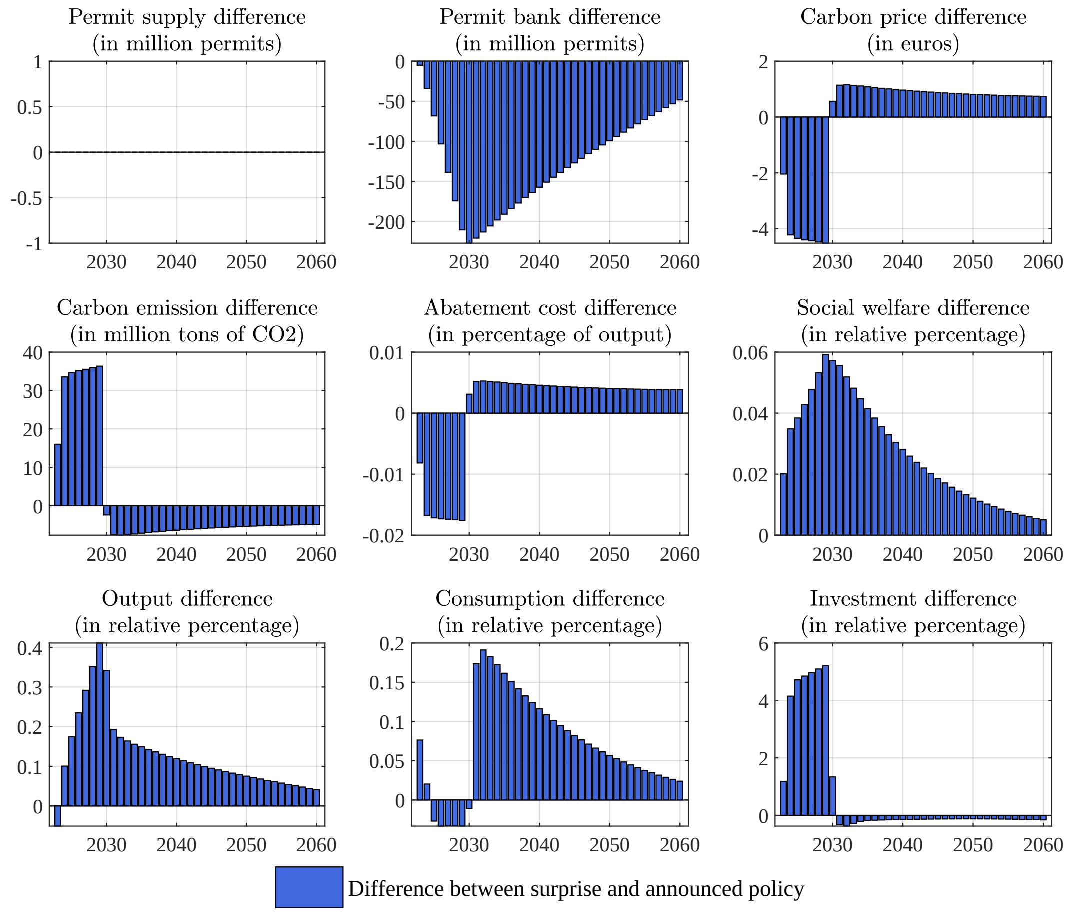


Note: This figure displays the trajectories of the main environmental and macroeconomic variables by applying alternative cap scenarios (Alternative I sets a LFR at 2.2% after 2031 and Alternative II sets a LFR at 10% after 2031). Variables are expressed in monthly terms. Macroeconomic variables are detrended. Permit supply is assumed to be equal to the cap policy.

not because the permit supply is zero that the emissions are. In fact, stored permits lead to pollution in the future. It would take a much greater drop in supply to be close to net zero by 2050.

Thus, we understand from these simulations that not only does the amount of an announced regulatory change matter when firms can bank carbon emission allowances, but also the timing of this announcement. Although changes are typically announced years in advance, they can also be rather short notice, as was the case with the 2024 cap decrease of 90 million allowances. To assess the role of expectations in shaping policy outcomes, we now compare the pre-announced "Alternative II" scenario with a surprise "Alternative II" scenario. In the latter case, agents believe that they are on the track of the baseline scenario until 2030.

FIGURE 10. The effects of the timing of a policy announcement



Note: This figure displays the differences (expressed in level or relative percentage) between the main variables' trajectories resulting from a surprise cap reduction policy and those resulting from an announced policy. Each blue bar represents the difference over the year. Permit supply is assumed to be equal to the cap policy.

At that point, they are surprised with an announcement, informing them of the faster cap decrease to come from 2031 on. Figure 10 shows the differences between the two paths, taking the surprise scenario minus the announced scenario.

We see that the effects at the time of policy implementation are rather similar, regardless of how the agents learn about it (announced or surprised). However, significant differences are observed during the pre-implementation period. This means that surprising agents would entail gains in aggregate demand (mainly through investment) until 2029. These economic gains stem from inaction until the policy is implemented, allowing firms to act less against climate change. The accumulation of extra capital allowed by a lower carbon price and abatement efforts before the policy announcement translates into long lasting small gains in GDP, consumption and welfare. But these gains must also be rebalanced against the possible climate-related costs from an increased stock of carbon as emissions continue to grow.

Thus, this exercise underlines the importance of an environmental policy announcement to act on the behavior of firms and thus reduce emissions as soon as it is known and not upon its actual implementation.

In summary:

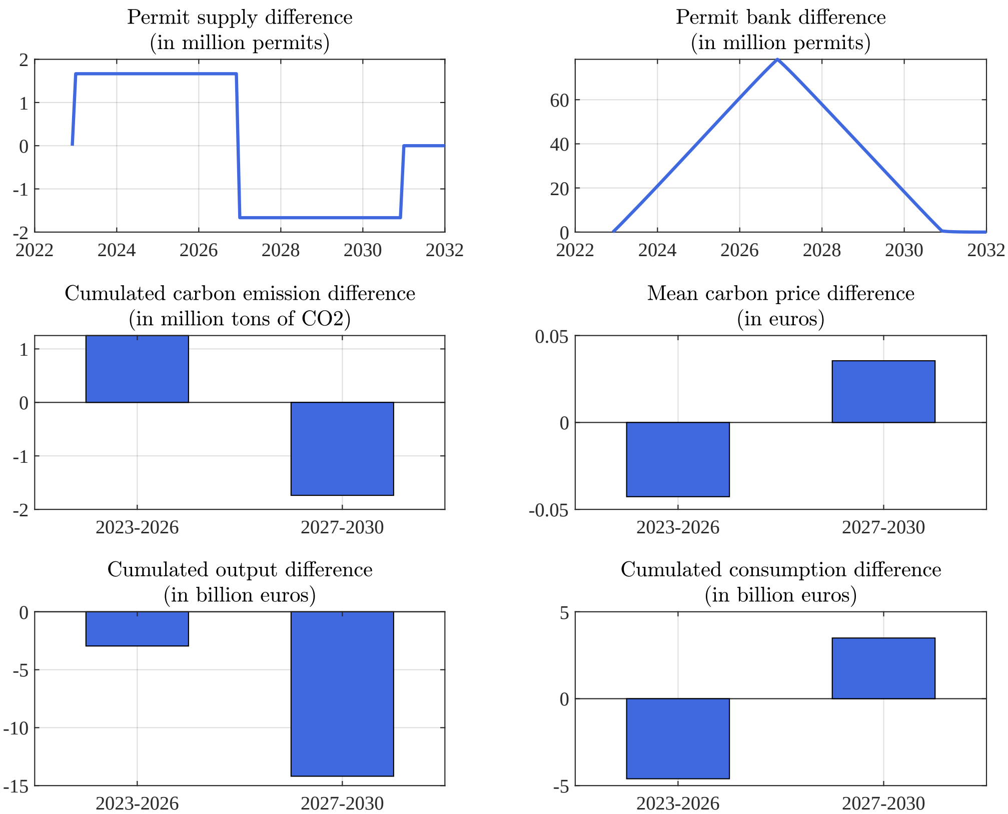
Result 4. *Announcing a policy in advance allows agents to modify their behavior accordingly, thus reducing emissions from the day of the announcement and not at the time of its implementation.*

5.4. **Carbon permit supply frontloading.** On February 21, 2023, the European Parliament formally adopted an amending regulation (Regulation (EU) 2023/435) to include chapters of the European Commission's REPowerEU plan in the Recovery and Resilience Facility.[5] The purpose is to increase the resilience, security and sustainability of the Union's energy system through a decrease in the dependence on fossil fuels and a diversification of energy supplies. This initiative seeks to boost the roll-out of renewables by increasing the bloc's target from 40% to 45% of the total energy supply by 2030. One of the sources to support these measures is the Emission Trading System, with 20 billion euros coming from the auction of ETS allowances. Eight of the 20 billion will come from the frontloading of the allowances. Indeed, from 2023 to August 2026, a number of allowances from the quantity that would otherwise be auctioned from January 2027 to December 2030 will be auctioned until the revenue obtained reaches 8 billion euros. In principle, the allowances should be auctioned in equal annual volumes over the 2023-2026 period. To quantitatively assess the economic impact of such frontloading, we explicitly modify the baseline scenario of Subsection 5.1. Figure 11 displays the differences between the baseline scenario and the case with frontloading for the relevant variables.

Providing more permits at first naturally increases the level of emissions by 1.2 MtCO2e over the period 2023-2026 and then removing it leads to a drop of 1.6 MtCO2e over the 2027-2030 period. We might think that there is a total gain in terms of emissions but we remember once again that what accumulates in the atmosphere cannot be removed. In other words, the asymmetric nature of the stock of atmospheric pollution implies that (i) emitting emissions fuels it with certainty, but (ii) a reduction in emissions does not automatically reduce it. Moreover, the "saved" 0.4 MtCO2e will be emitted anyway after 2030 because the total permit supply remains unchanged. Given the temporary nature of this fontloading, firms increase

[5]The aim of the Recovery and Resilience Facility is to mitigate the economic and social impact of the coronavirus pandemic and make European economies and societies more sustainable, resilient and better prepared for the challenges and opportunities of green and digital transitions

FIGURE 11. The impacts of fontloading allowances



Note: This figure displays the differences due to a frontloading policy over its implementation phases for relevant variables (frontloading versus baseline scenarios).

their bank of permits during the first period to use it in the second period. The overall effect on carbon price is slightly negative. The price drops when the permit supply increases and vice versa. However the magnitude of these movements is limited because banking opportunities act as stabilizers. The macroeconomic effects are generally negative. Despite a higher permit supply, consumption falls during the first period. This is because firms devote resources to buying extra permits without using all of them to immediately support production. Later, when firms use permits that they have already paid for and saved in the bank, consumption is allowed to increase. The net effect on consumption during the two periods is slightly negative. GDP falls in both periods, and more sharply in the second period due to a drastic reduction in investment.

Result 5. *Frontloading permit allowances results in (i) a net drop in emissions but after an increase in the stock of pollution in the atmosphere, (ii) a net negative effect on the carbon price and (iii) a reduction in GDP over both periods of frontloading and withdrawal.*

5.5. **The market stability reserve.** In 2014, amid the built-up of a surplus of allowances in circulation that started in 2009 (see Figure 1), the European Commission postponed the auctioning of some allowances. The surplus, or bank, amounted to over 2 billion allowances at the start of Phase 3 of the EU-ETS. This is, in part, owed to the financial crisis and remains substantial until this day. A large surplus threatens the ETS functioning in several ways. It reduces the short-term demand for newly issued allowances, thus reducing the carbon price and incentives for firms to engage in a green transition. In the long term, it can also affect the ability of the ETS to meet more demanding emission reduction targets cost-effectively (see the European Commission's dedicated webpage). Concerns that the surplus would remain over 2 billion allowances for a decade or more, despite the increase in the LRF, urged the Commission to react. In fact, the bank of allowances was still at almost 1.5 billion in 2021 and our previous subsections predict that, in the absence of an adjustment mechanism, it could eventually amount to over 6 billion.

In an attempt to tackle structural supply-demand imbalances, the European Parliament thus postponed the auction of 900 million allowances over the 2014-2016 period. This was meant as a short-term solution as Decision (EU) 2015/1814 introduced the market stability reserve (MSR) to be implemented at the beginning of 2019. It functions by triggering adjustments to annual auction volumes if the requirements based on the level of the aggregate bank of allowances are met. For this purpose, the European Commission has begun publishing the total number of allowances in circulation (TNAC) annually. When TNAC is above a certain threshold, the quantity of allowances that should have been auctioned during the next 12 months, calculated as a percentage of TNAC, is instead placed in the reserve. In contrast, when TNAC is below a certain threshold, allowances are released from the reserve and auctioned off. Any allowances placed in the MSR above a certain threshold are cancelled. The 900 million allowances postponed in 2014-2016 were placed in the reserve instead of being auctioned in 2019-2020, as initially planned. In its most recent version, after amendments announced in Directive (EU) 2018/410 and Directive (EU) 2023/959 of the European Parliament, the MSR works in the following way. If TNAC is between 833 million and 1,096 million, the difference between TNAC and 833 million is transferred to the reserve. If TNAC is above 1,096 million, the number of allowances to be placed in the reserve amounts to 24% of TNAC. This percentage should go (back) down to 12% after 2030. If TNAC is less than 400 million, 100 million allowances should be released from the reserve and auctioned off (if there are less than 100 million allowance in the reserve, they should all be released). In addition, any allowances held in the reserve above 400 million are cancelled.

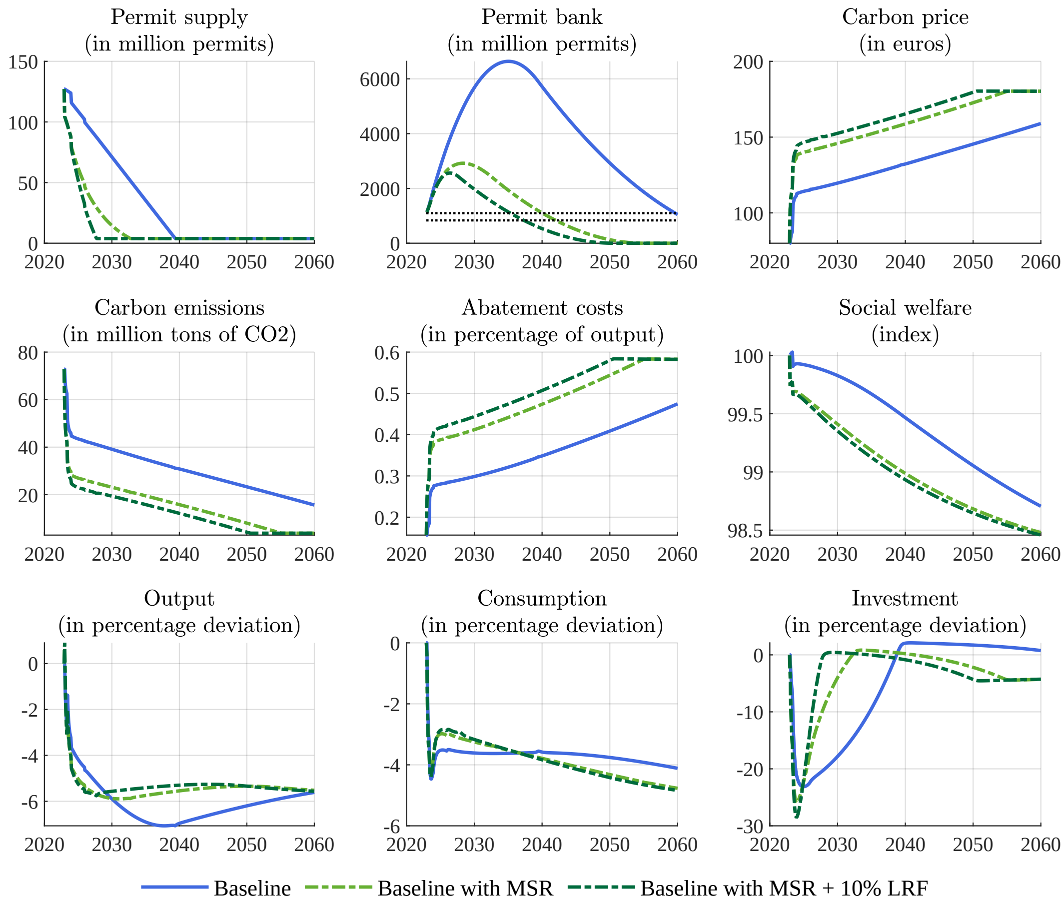
In our framework, TNAC is represented by the firms' bank of permits. To incorporate the adjustments in the supply of permits due to the MSR in our simulations, we modify Equation (21) accordingly:

$$\vartheta_t = \varepsilon_{\vartheta,t}\bar{\vartheta} - \mathbb{1}_{\{(b_t>\underline{b})\cap(b_t<\bar{b})\}}\frac{b_t-\underline{b}}{12} - \mathbb{1}_{\{b_t>\bar{b}\}}\tau\frac{b_t}{12} \tag{24}$$

where $\mathbb{1}\{\cdot\}$ is the indicator function, $\underline{b} = 833$ and $\bar{b} = 1096$ are the first and second thresholds on the bank (in million allowances), respectively; and $\tau = 24\%$ is the percentage of allowances in the bank removed from the supply above the second threshold. We divide both terms related to the adjustments by 12, because our model is at a monthly frequency. Note that for computational reasons, we do not consider allowances released from the MSR when TNAC is below 400 million. However, there is little quantitative difference because in our perfect foresight setup, once the reserve starts to be depleted, it is not filled again. Hence, introducing this feature would only increase the permit supply by 400 million allowances over the course of four years. Moreover, even with the MSR, TNAC is not expected to return to 400 million in the coming years, leaving time for changes in regulations. Not considering these aspects allows us not to have a state variable that tracks the number of permits in the reserve and requires fewer conditional statements.

The results are shown in Figure 12. Introducing the MSR further reduces the supply of permits and increases the price of carbon, which is expected to reach 180 euros by 2060. Consequently, the MSR eats away the bank, and it takes approximately 17 years for banking to fall below intake threshold $\underline{b}$, in line with Quemin and Trotignon (2021). The emissions, carbon price and abatement costs all reach a plateau once both the new permit supply and the bank of permits reaches zero. During the transition, GDP falls on average less in the presence of the MSR than in the baseline case (5.3% vs. 6%), due to faster recovery of investment but also an increased cost component. However, it reaches the same level in 2060. Consumption is initially relatively higher in the presence of the MSR, despite the decreased permit supply, and increased carbon price and abatement effort. This is because firms subjected to the MSR drastically reduce their bank intake rate from the start, directly using a larger proportion of permits at the time they buy them. By 2030, firms are already using stored (and hence paid) permits. However, by the time firms subjected to the baseline cap begin depleting their own permit banks, consumption in the MSR case falls gradually and durably below. By

FIGURE 12. Permit banking and the market stability reserve



Note: This figure displays the trajectories of the main environmental and macroeconomic variables under alternative cap scenarios. Variables are expressed in monthly terms. Macroeconomic variables are detrended. Permit supply is assumed to be equal to the cap policy.

2060, consumption has fallen by almost 5% in the MSR case versus 4% in the baseline case, making the welfare index to be always lower with the MSR. The combination of the MSR and LRF trajectories announced in 2023 makes it possible to achieve net-zero emissions slightly beyond the date planned by the Paris Agreement (i.e., in 2053 instead of 2050). However, increasing the LRF to 10% from 2024 onwards (dark green dotted line) would save these three years without deleterious macroeconomic effects. In this case, the bank of permits is reduced more quickly and will reach the MSR intake threshold in 2035.

Result 6. *The market stability reserve is a powerful tool to slow down firms' banking of permits and thus reduce emissions more quickly. By combining it with a higher LRF (e.g, 10%), the net-zero objective would be achieved in 2050, with an average GDP cost of approximately 5.3% and an average consumption cost of approximately 3.9%.*

6. CONCLUSION

This study investigates the general equilibrium effects of permit banking during the transition to a low-carbon economy. We develop and estimate an E–RBC model incorporating an ETS market in which firms can store permits but are not allowed to borrow them. We implement recent decisions by the European Parliament and examine the nonlinear dynamics between environmental and macroeconomic variables, through projection exercises up to 2060.

We find that the EU-ETS 2023 cap reform, defined as a new sequence of linear reduction factors of the cap for stationary installations (4.3% from 2024 to 2027 and 4.4% from 2028 onward), would lead to a strong increase in permit banking until 2035, a doubling of the carbon price, and an average GDP loss of 6% by 2060. The market stability reserve, which triggers adjustments to annual auction volumes if requirements based on the level of the aggregate bank of allowances are met, is a powerful tool that slows down firms' banking of permits and reduces emissions more quickly. By combining it with a higher LRF (e.g, 10%), the net-zero objective would be achieved in 2050, with an average GDP cost of approximately 5.3%. Announcing a policy in advance allows agents to modify their behavior accordingly, thus reducing emissions from the day of the announcement and not at the time of its implementation. Importantly, forgetting permit banking when assessing cap policies would lead to both a significant underestimation of the total macroeconomic effects and an incorrect carbon emission path.

Our new estimated model contributes to the literature by quantifying the role of permit banking and its interaction with cap policies. Nevertheless, its structure can be extended in several dimensions, which represent interesting research avenues. For instance, it was assumed that the EU-ETS market was applied to all companies in the economy. However, it covers approximately 40% of total EU's greenhouse gas emissions and approximately 10,000 companies in the energy sector and manufacturing industry, as well as aircraft operators. Hence, it would be pertinent to categorize companies into two subsets: one subject to regulation and the other not, and examine its impact on GDP.

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Appendix A. Solution method

This appendix provides details of the model resolution method in the presence of occasionally-binding constraints.

Let us stack the endogenous variables into one vector:

$$z_t = [c_t, y_t, x_t, k_t, n_t, r_{k,t}, w_t, q_t, \mu_t, b_t, p_{e,t}, \lambda_{h,t}, \lambda_{f_1,t}, \lambda_{f_2,t}, \varepsilon_{a,t}, \varepsilon_{\mu,t}, \varepsilon_{\vartheta,t}]', \tag{A.1}$$

and all structural shocks in $\epsilon_t = [\zeta_{a,t}, \zeta_{\mu,t}, \zeta_{\vartheta,t}]'$.

The regime-switching model reads as:

$$\mathbb{1}_{b_t=0}\mathrm{E}_t\{f\left(z_{t+1}, z_t, z_{t-1}, \epsilon_t\right)\} + \mathbb{1}_{b_t\geq 0}\mathrm{E}_t\{f^*\left(z_{t+1}, z_t, z_{t-1}, \epsilon_t\right)\} = 0, \tag{A.2}$$

where $f(\cdot)$ is the system of equations under the normal regime, when all banking has been exhausted, and $f^*(\cdot)$ is the alternative regime when there is positive banking.

Consider a first-order Taylor expansion around the normal regime model (with $\bar{z}$ satisfying $f\left(\bar{z}, \bar{z}, \bar{z}, 0\right) = 0$), with endogenous variables denoted as $\hat{z}_t = z_t - \bar{z}$. The Taylor expansion of each regime $f(\cdot)$ and $f^*(\cdot)$ yields :

$$\begin{aligned} F\mathrm{E}_t\{\hat{z}_{t+1}\} + G\hat{z}_t + H\hat{z}_{t-1} + L\epsilon_t &= 0, & \text{(A.3)}\\ F^*\mathrm{E}_t\{\hat{z}_{t+1}\} + G^*\hat{z}_t + H^*\hat{z}_{t-1} + L^*\epsilon_t + \mho^* &= 0, & \text{(A.4)} \end{aligned}$$

where F, F^*, G, G^*, H, H^*, L, L^* are Jacobian matrices from each regime, while $\mho^*$ is a constant vector that accounts for the difference in the steady state across the two regimes (i.e., $f^*\left(\bar{z}, \bar{z}, \bar{z}, 0\right) \neq 0$).

The normal regime is assumed to be the baseline regime. The recursive solution of the problem around the normal regime is given by:

$$\hat{z}_t = P\hat{z}_{t-1} + Q\epsilon_t. \tag{A.5}$$

Note that if $\hat{b}_t$ is positive, one switches to the other regime. The tricky issue is how to deal with the expectation term $F^{*}\mathrm{E}_t\{\hat{z}_{t+1}\}$. Under rational expectations, agents take their decisions knowing how long the alternative regime will last, so how long $f^{*}(\cdot)$ applies. There is no closed-form expression to find the duration. The latter must be determined numerically by iterations.

The general formulation of the solution for any duration d in the alternative regime can be described by the following system:

$$\hat{z}_t = P(d)\,\hat{z}_{t-1} + Q(d)\,\epsilon_t + R(d)\,, \tag{A.6}$$

$$P(d) = \left[F^{*}P(d-1) + G^{*}\right]H^{*}, \tag{A.7}$$

$$Q(d) = \left[F^{*}P(d-1) + G^{*}\right]L^{*}, \tag{A.8}$$

$$R(d) = \left[F^{*}P(d-1) + G^{*}\right]\mho^{*}, \tag{A.9}$$

where $P(0) = P$, $Q(0) = Q$ and $R(0)$ is a vector of zeros.

Therefore, the solution of the model is state dependent. The duration of the alternative regime affects the propagation and strongly enriches the dynamics of the model. At the same time, a drawback is that we must guess the duration. Guerrieri and Iacoviello (2015) discuss one possible solution which they call the guess-and-try algorithm. Note also that the duration d must be finite in order to solve numerically the problem, put differently dynamics of the model must go back to the normal regime.

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